

ELEN E3106/4106 Lecture 18

BJTs Part IV: Models and Secondary Effects

Outline

- Ebers-Moll model
- Drift Transistors
- Total Transit time & Kirk Effect
- Small-signal mode & capacitance
- Cut-off Frequency
- Charge control model

Assignments:

Reading: C. Hu §8.6-8.10

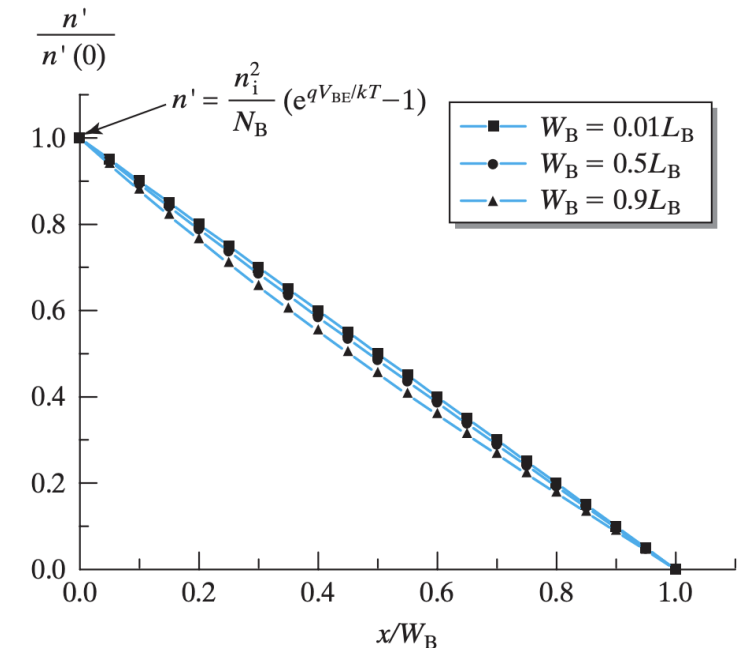
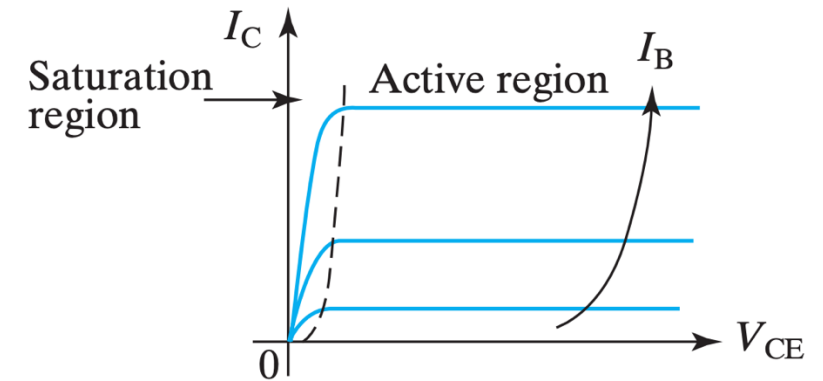
Homework 7 due Fri. Nov. 14th by 5pm

Recap on BJT types

- It's important to remember that *forward biased* p-n junctions have a more positive voltage on the _____ relative to the _____
- In *reverse bias*, the _____ is more positive relative to the _____
- Generally, voltage subscripts are denoted p to n side (_____)
 - For npn, the E and C are n-type and B is p-type so V_{pn} would be V_{BE} and V_{BC}
 - For pnp, the E and C are p-type and B is n-type V_{pn} would be V_{EB} and V_{CB}
- Ex. in _____ mode, the EB junction is forward biased and the CB is reverse biased *regardless of BJT type*
 - In npn, $V_{BE} > 0$ and $V_{BC} < 0$
 - In pnp, $V_{EB} > 0$ and $V_{CB} < 0$
- The total voltage from collector to emitter is just the sum of the voltages across the 2 junctions: $V_{CE} = V_{CB} + V_{__}$
 - But we careful again with voltage polarity!
 - Ex. We have an npn in saturation mode (both junctions FB) with $V_{BE} = 0.8\text{ V}$, $V_{BC} = 0.6\text{ V}$, hence $V_{CE} = 0.8 - 0.6\text{ V} = 0.2\text{ V}$

A Note on Saturation and Linear Regimes

- Let's take another look at why I_C decreases as $V_{CE} \rightarrow 0$ in the saturation regime
 - Both junctions are _____. Increasing FB V_{BC} decreases V_{CE} for a given constant V_{BE} ($V_{CE} = V_{CB} + V_{BE}$)
 - In an npn, when BC is FB, the excess e- ($\Delta n_p = n'$) at $x = W_B$ rises as the collector also injects e- into the base
 - _____ is depressed, and therefore I_C is depressed
- The dashed line boundary between the transistor saturation and active modes represents where $V_{BC} = \underline{\hspace{2cm}}$
 - E.g. the CB junction goes from FB to RB
- And why is I_C practically flat in active mode?
 - Recall that I_C is essentially independent of V_{CB} , as long as V_{CB} is RB
 - Instead, I_C depends on V_{BE} . So even as we increase RB V_{BC} and V_{CE} increases for a given V_{BE} ($V_{CE} = V_{CB} + V_{BE}$), I_C remains the same



Ebers-Moll Model

- A popular model (frequently used in SPICE circuit simulation) that mathematically describes the BJT

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- We will look at saturation and active modes

- Let's first assume that $V_{BC} = 0$ in an npn. Our current equations become:

$$I_C = I_S(e^{qV_{BE}/kT} - 1)$$

$$I_B = \frac{I_S}{\beta_F}(e^{qV_{BE}/kT} - 1)$$

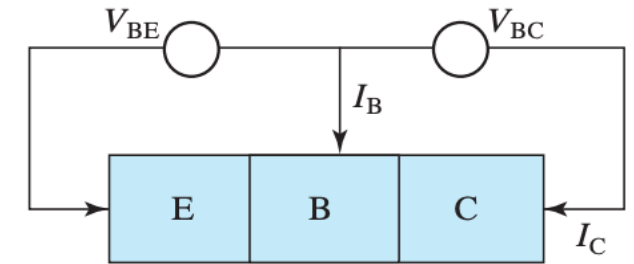
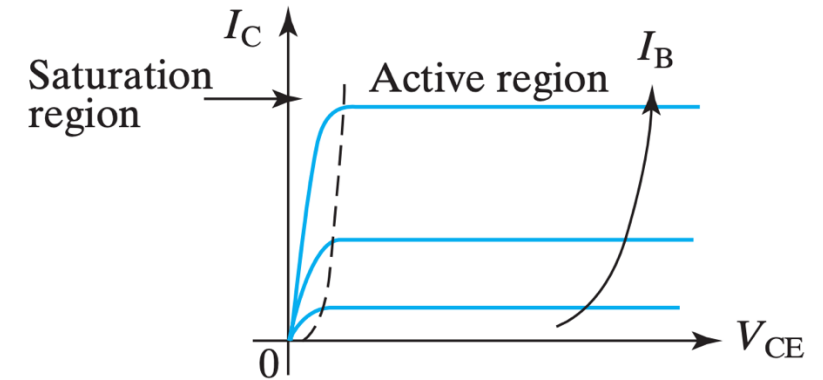
- Now let's assume the opposite ($V_{BE} = ______$):

- e- are injected from C \rightarrow B and flow to $______$

$$I_E = I_S(e^{qV_{BC}/kT} - 1)$$

$$I_B = \frac{I_S}{\beta_R}(e^{qV_{BC}/kT} - 1)$$

$$I_C = -I_E - I_B = -I_S\left(1 + \frac{1}{\beta_R}\right)(e^{qV_{BC}/kT} - 1)$$



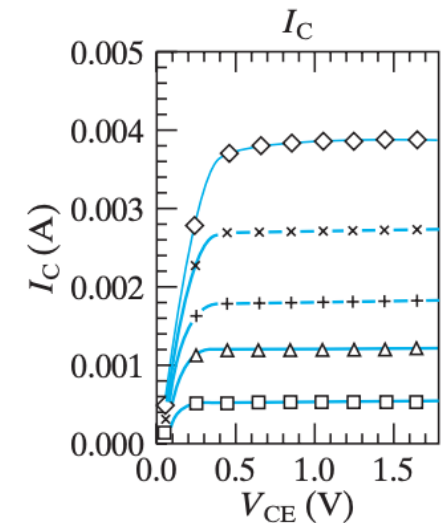
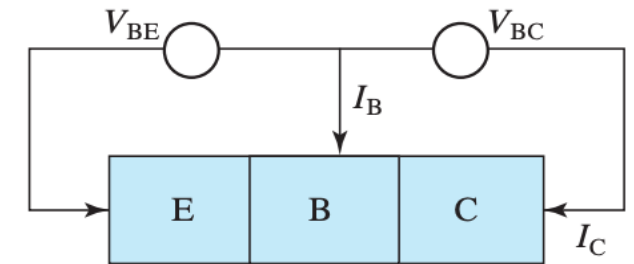
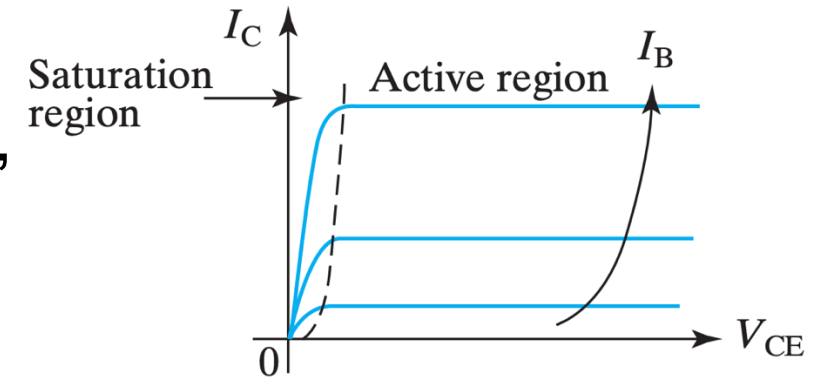
Ebers-Moll Model

- When both $V_{BC}, V_{BE} \neq 0$, our eq's are superimposed, comprising the Ebers-Moll model:

$$I_C = I_S(e^{qV_{BE}/kT} - 1) - I_S\left(1 + \frac{1}{\beta_R}\right)(e^{qV_{BC}/kT} - 1)$$

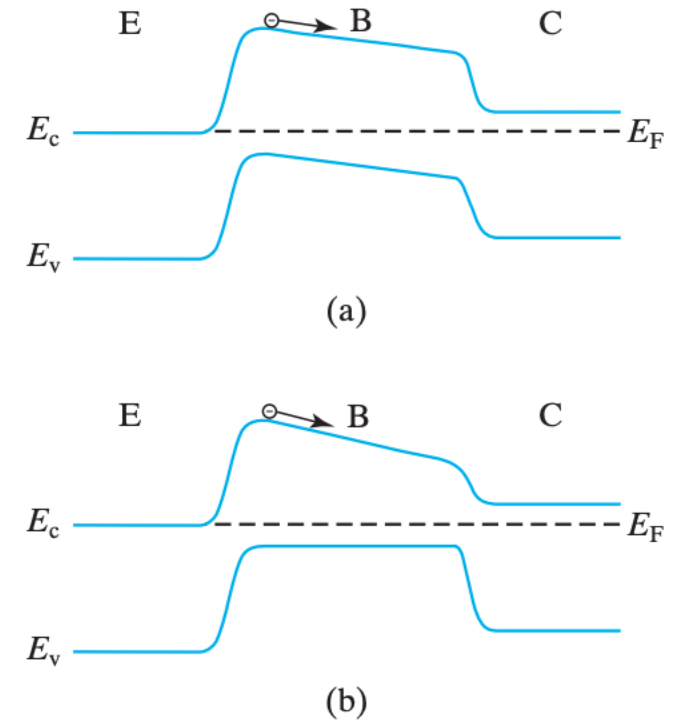
$$I_B = \frac{I_S}{\beta_F}(e^{qV_{BE}/kT} - 1) + \frac{I_S}{\beta_R}(e^{qV_{BC}/kT} - 1)$$

- The model (lines) agrees with experimentally measured data (_____) in both the saturation and linear regimes. Example shown is for an SiGe-base HBT.



Drift Transistors: Built-In Base Fields

- How else can we reduce base transit time and improve speed?
 - Recall: we discussed compositionally _____ the base to establish a _____ that aids the flow of carriers from emitter to collector
- Alternatively, we could create a built-in base field by varying the doping concentration, _____, through the base
- The varying N_B creates $\frac{dE_v}{dx}$ and $\frac{dE_c}{dx}$, and the field
- In this case, the base transit time is NOT reliant on diffusion ($\tau_t \neq \frac{W_B^2}{2D_B}$) but instead reliant on _____!
- We can achieve τ_t _____ $\frac{W_B^2}{2D_B}$
- These BJTs are called _____



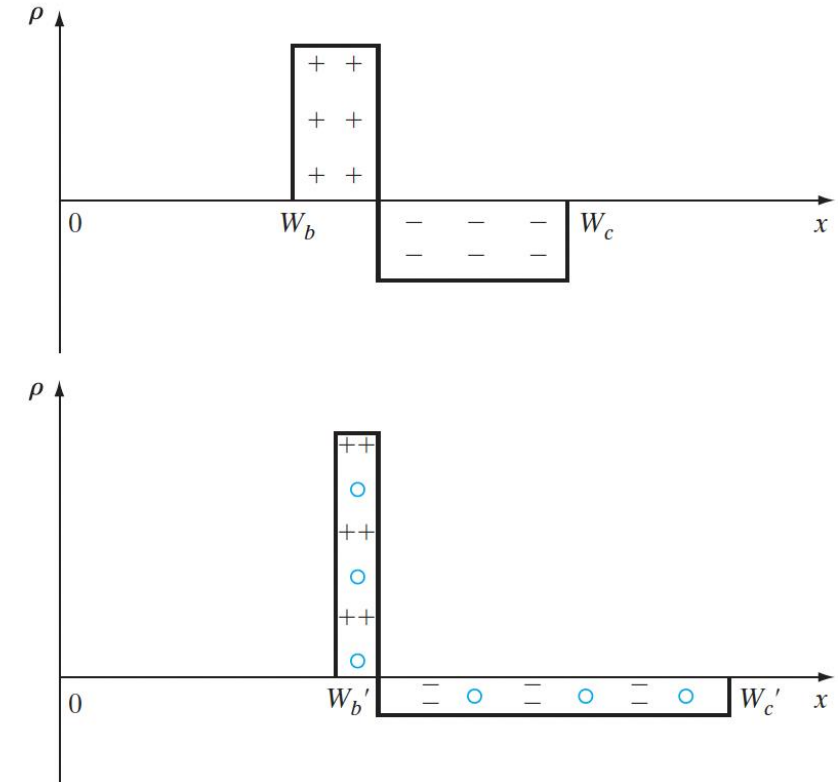
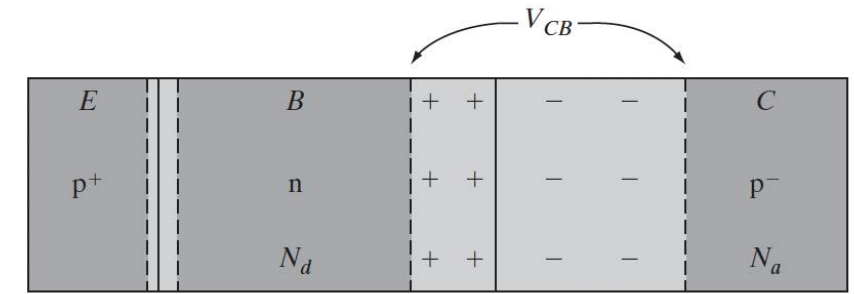
Sources: C. Hu *Modern Semiconductor Devices for Integrated Circuits*

Emitter-to-Collector Transit Time

- What's the total time it takes for carries to cross from E -> C?
 - _____, total forward transit time (Units: s)
- What portion of τ_F is made up by the base transit time, τ_t ?
- What else contributes to transit (storage) time?
- As I_C increases, τ_F increases. Why?
 - The dopant density (_____ = _____) is insufficient to support the collector current, even when carriers move at saturation velocity (_____)

Kirk Effect

- Effective _____ of the base region at high I_C !
- Also causes current gain drop at high collector currents!
- Reverse biased CB junction has a build up of mobile carriers due to strong collector current
- Let's take a pnp BJT. The injected h^+ mobile charges add to the fixed N_d^+ on the _____ side of the CB depletion region, and _____ from the fixed N_a^- on the collector side
- Fewer uncompensated donors (and thus a smaller depletion width) are needed to maintain the reverse voltage
- The base width increases from _____ to _____



Kirk Effect

- Remember: the depletion region extends more into the more lightly doped (collector) side
- This is equivalent to moving the base-collector junction deeper into the collector
- Drops the current gain due to an increase of the base transit time!
- With even higher current levels and base widening, what effect can occur in the CB junction?

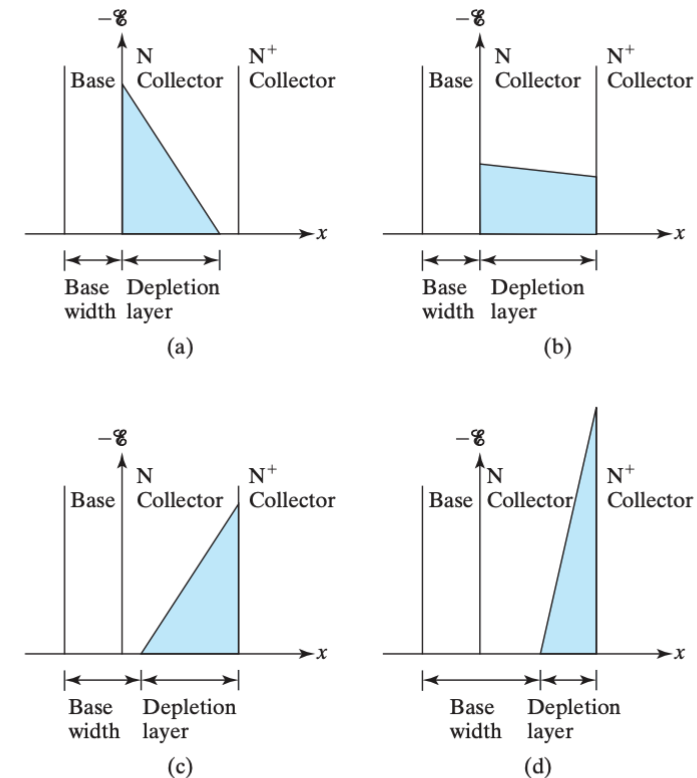
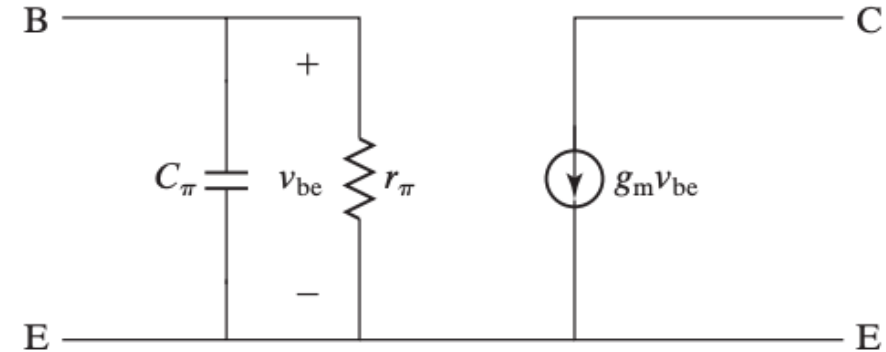


FIGURE 8-18 Electric field $E(x)$, location of the depletion layer, and base width at (a) low I_C such as $0.1 \text{ mA}/\mu\text{m}^2$ in Fig. 8-17; (b) larger I_C ; (c) even larger I_C (such as $1 \text{ mA}/\mu\text{m}^2$) and base widening is evident; and (d) very large I_C with severe base widening.

Small-Signal Model

- Small-signal models apply when we have an _____ that is _____ relative to the DC bias currents and voltages.
 - Useful for analog circuits!
 - These models are AC equivalent circuits where nonlinear circuit elements are replaced by linear elements whose values are _____
- Ex: Let's say we have a small input signal ($v_{BE} \sim 10$ mV sinusoidal), superimposed on a DC bias
- Which configuration is this? Common _____



Small-Signal Model and Transconductance

- Assuming V_{BE} is not near zero, we can neglect the “1” term:

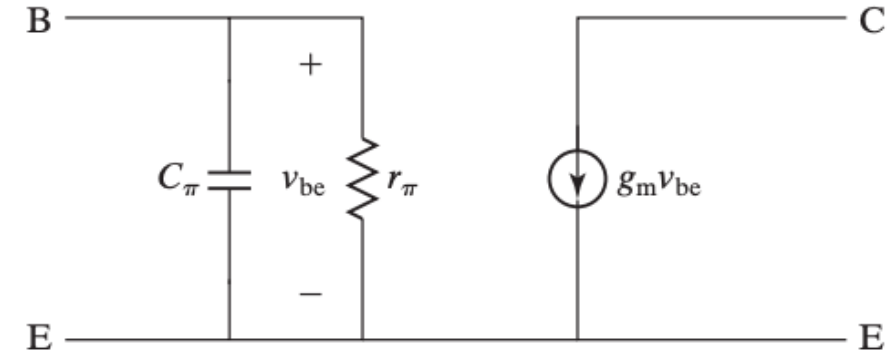
$$I_C = I_S(e^{\frac{qV_{EB}}{kT}} - 1)$$

- The BJT *transconductance*, _____, is the change in current with the change in voltage:

$$\begin{aligned} g_m &\equiv \frac{dI_C}{dV_{BE}} = \frac{d}{dV_{BE}}(I_S e^{qV_{BE}/kT}) \\ &= \frac{q}{kT} I_S e^{qV_{BE}/kT} = I_C / \frac{kT}{q} \end{aligned}$$

$$g_m = I_C / \frac{kT}{q}$$

- So what's g_m at 300 K?
- In general, we want to _____ g_m



Small-Signal Model and Charge-Storage Capacitance

- The input (base) appears as a parallel RC circuit with

$$\frac{1}{r_\pi} = \frac{dI_B}{dV_{BE}} = \frac{1}{\beta_F} \frac{dI_C}{dV_{BE}} = \frac{g_m}{\beta_F}$$

$$r_\pi = \beta_F / g_m$$

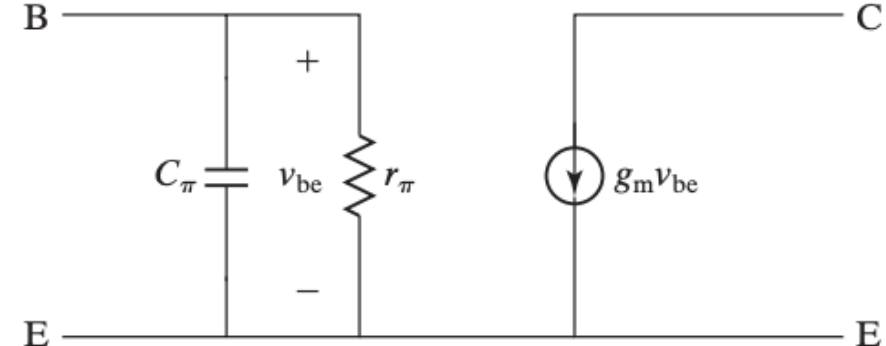
- _____ is the excess carrier charge stored in BJT
 - If we have $Q_F = 1 \text{ pC}$, we have _____ of h^+ and _____ of e^-
 - In npn, excess h^+ charge is supplied by _____

- So, the base presents this capacitance to the input:

$$C_\pi = \frac{dQ_F}{dV_{BE}} = \frac{d}{dV_{BE}} \tau_F I_C = \tau_F g_m$$

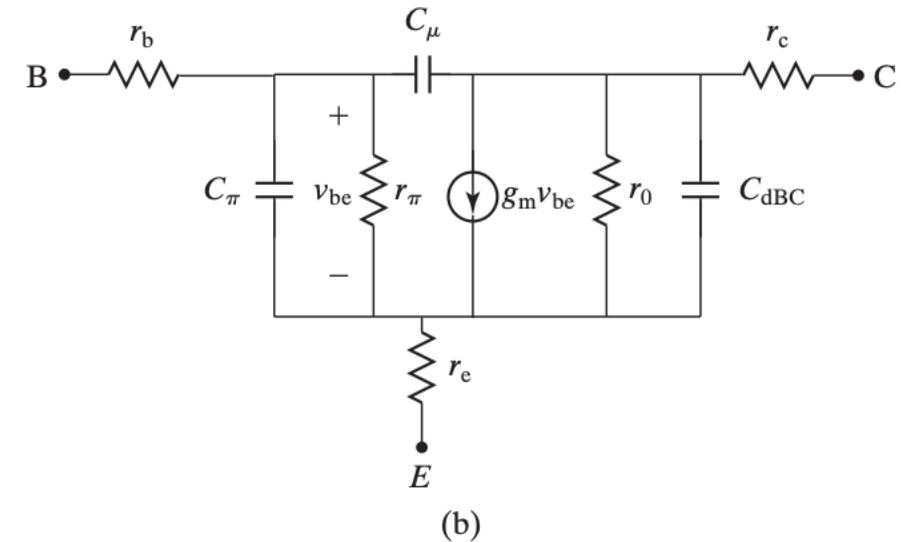
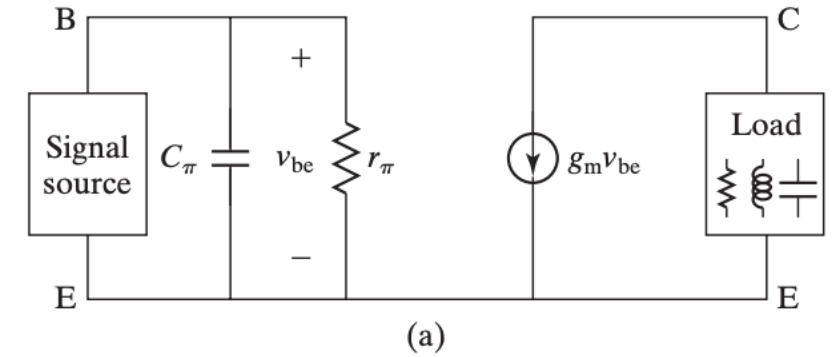
- This *charge-storage capacitance* (diffusion capacitance) must also take into account the EB junc. depletion-layer charge, C_{dBE}

$$C_\pi = \tau_F g_m + C_{dBE}$$



Small-Signal Model

- What's the significance of the small-signal model?
- Now we can analyze circuits with arbitrary signal-source and load impedance network by hand!
- This simple small-signal model can be solved to find g_m , r_π , C_π
- Or, SPICE circuit simulations can be used with a more accurate small-signal model (bottom)
 - Includes effects of base width modulation, etc.



Cut-Off Frequency

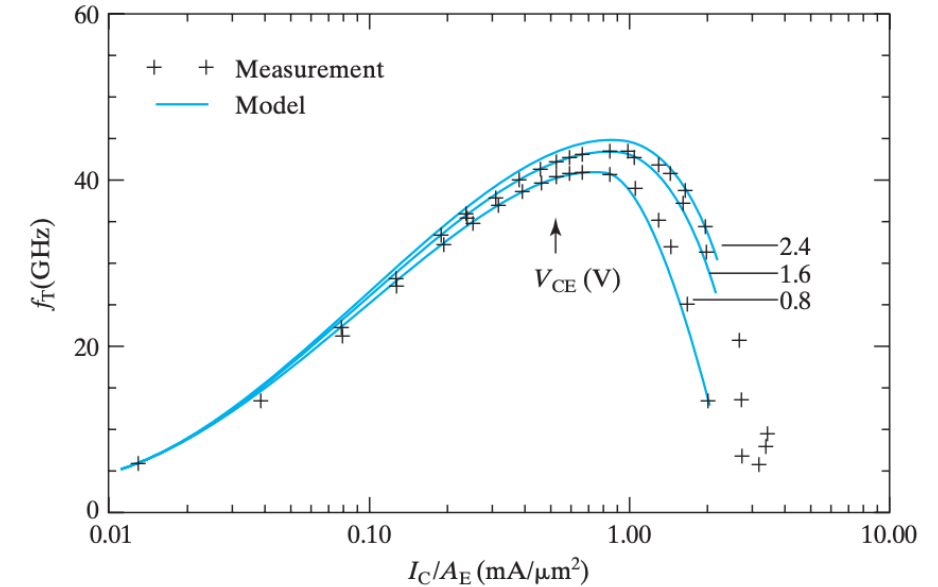
- _____ is the *cut-off frequency*, commonly used parameter to compare the speed of transistors. A more complete small-signal model yields:

$$f_T = \frac{1}{2\pi[\tau_F + (C_{dBE} + C_{dBC})kT/(qI_C) + C_{dBC}(r_e + r_c)]}$$

- Why? It can be shown that the frequency of *unity power gain* (β falls to 1, max. oscillation frequency) is given,

$$f_{\max} = \left(\frac{f_T}{8\pi r_b C_{dBC}} \right)^{1/2}$$

- f_T rises with increasing I_C due to increasing g_m
- At very high I_C , τ_F increases due to base widening and f_T decreases
- So where do we bias BJT the best high-frequency performance?

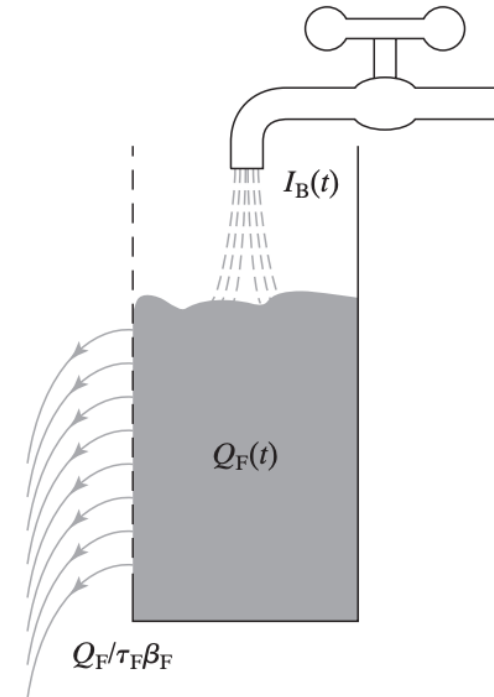


Charge Control Model

- What if the input signal is large? We can't use small signal model. What $I_C(t)$ is produced by a time-varying $I_B(t)$?
- Even if the charge storage varies with time, the relation still holds true:

$$I_C(t) = Q_F(t) / \tau_F$$

- This suggests I_C is controlled by Q_F , hence the name *charge control model*. In DC, I_B and Q_F : $I_B = I_C / \beta_F = Q_F / \tau_F \beta_F$
- AKA In order to sustain a constant excess hole charge, h^+ must be supplied to the transistor through I_B to replenish the h^+ lost to recombination.
- Analogy would be filling a leaky bucket from a faucet, where the amount of water $Q_F(t)$ rises or falls at the rate of supply $I_B(t)$ minus leakage $Q_F / (\tau_F \beta_F)$



$$\frac{dQ_F}{dt} = I_B(t) - \frac{Q_F}{\tau_F \beta_F}$$